A Configuration Oriented SPICE Model for Multiconductor Transmission Lines in an Inhomogeneous Medium

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***Abstract*— A configuration oriented Spice model for multiple coupled lines in an inhomogeneous medium is presented. The model is useful for modelling general single and multilayer coupled line structure. The system consists of a network of uncoupled transmission lines and is readily modelled with simulation tools like Libra and SPICE and provides an equivalent circuit representation which is simpler and more versatile as compared to the model based on modal decomposition.**

I. Introduction

The equivalent circuit models for multiconductor transmission line systems in an inhomogeneous medium have been based on the decomposition of the signal into normal modes of propagation. These modes in general propagate with different velocities and the resulting SPICE models incorporate this modal decomposition via a system of transmission lines connected to a network of linear dependent sources[1]. Simplified versions of this model valid for special cases of homogeneous medium and electrically identical lines have also been proposed [2,3]. Recently, a simpler configuration oriented equivalent circuit model, consisting of a system of transmission lines only, was reported for the case of homogeneous medium[4]. Similar approximate models valid for special cases of inhomogeneous structures have also been proposed [5,6]. In this paper, the configuration oriented SPICE model for the general case of inhomogeneous multilayer multiconductor structure is presented. In addition to having a simpler SPICE input data requirements as compared to the modal decomposition based model, the configuration-oriented model is readily implemented for the simulation of multiconductor lossy, dispersive inhomogeneous structures.

II. Equivalent circuit model

The configuration oriented models are based on the equivalent circuit realization of the admittance or the impedance matrix of the n-coupled line $2n$-port. For example, the admittance matrix of the $2n$-port can be expressed as [7],

$$[Y] = \begin{bmatrix} [Y_A] & [Y_B] \\ [Y_B] & [Y_A] \end{bmatrix}$$

where

$$[Y_A] = [Y_{lm}] * [M_V][coth(\gamma_i l)]_{diag}[M_I]^T$$

$$[Y_B] = [Y_{lm}] * [M_V][csch(\gamma_i l)]_{diag}[M_I]^T$$

where the elements of $[Y_{lm}]$ are the line mode admittances, $[M_V]$ and $[M_I]$ are the n x n voltage and current eigenvector matrices and γ_i is the i th normal mode propagation constant and l is the length of the uniformly coupled multiconductor system. The above admittance matrix can in general be decomposed as,

$$[Y] = \sum_{m=1}^{n} [Y_m]$$

where $[Y_m]$ represents the partial admittance matrix of the $2n$-port which can be synthesized as a configuration oriented model for a homogeneous medium corresponding to the m th mode eigenvalue. Then the complete network is obtained as a parallel combination of the n, $2n$ ports each corresponding to an orthogonal mode. A similar procedure can be applied to the impedance matrix leading to a network of transmission lines that is equivalent to the multiconductor multiport. It is observed that admittance or impedance matrix of general n multiconductor lines can in general be simulated by $n^2(n+1)/2$ transmission lines. In case of symmetry the number of lines are reduced depending upon type of symmetry.

III. Network model for uniformly coupled line

The immittance matrix of the general asymmetric coupled line four port are given in [8]. These are readily decomposed into two modes

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$$[Y]_{4\times 4} = [Y_c]_{4\times 4} + [Y_\pi]_{4\times 4} \quad (1)$$

Each matrix can be modelled in general by three transmission lines connected in π configuration. These two networks connected in parallel give complete equivalent circuit as shown in Figure 1 a.

An alternate approach to deriving these circuits is with the help of characteristic immittance matrix of the coupled system. These matrices represent a network which terminates all the modes simultaneously on all the lines. The elements of these matrices represent the characteristic immittance of the transmission lines that constitute the equivalent circuit. For example, the characteristic admittance and impedance matrices for the coupled two line structure are given as,

$$[Y_c] = \begin{bmatrix} \frac{-R_\pi Y_{c1} + R_c Y_{\pi 1}}{R_c - R_\pi} & \frac{Y_{c1} - Y_{\pi 1}}{R_c - R_\pi} \\ \frac{Y_{\pi 2} - Y_{c2}}{(R_c - R_\pi)/R_c R_\pi} & \frac{R_c Y_{c2} - R_\pi Y_{\pi 2}}{R_c - R_\pi} \end{bmatrix} \quad (2)$$

$$[Z_c] = \begin{bmatrix} \frac{Z_{\pi 1} R_c - Z_{c1} R_\pi}{R_c - R_\pi} & \frac{Z_{c1} - Z_{\pi 1}}{(R_\pi - R_c)/R_c R_\pi} \\ \frac{Z_{c2} - Z_{\pi 2}}{R_c - R_\pi} & \frac{R_c Z_{c2} - R_\pi Z_{\pi 2}}{R_c - R_\pi} \end{bmatrix} \quad (3)$$

Where γ_c and γ_π are propagation constant, R_c and R_π are the ratio of voltage on conductor 2 to voltage on conductor 1 for the two modes, $Y_{c1}, Y_{c2}, Y_{\pi 1}, Y_{\pi 2}$ are corresponding line mode admittances and $Z_{c1}, Z_{c2}, Z_{\pi 1}, Z_{\pi 2}$ are line mode immittances.

The above admittance or impedance matrix for two coupled lines can be expressed as sum of two matrices, with π mode and c modes terms seperately which can be synthesized with six transmission lines as shown in Figure 1 a,b. For symmetric case ($R_c = 1$, $R_\pi = -1$ $Y_{c1} = Y_{c2}$ and $Y_{\pi 1} = Y_{\pi 2}$), the model reduces to four transmission line system as π network in [5] and T type system as shown in Figure 2 b. Similarly for symmetric coupled lines in homogeneous ($\gamma_c = \gamma_\pi$) medium the equivalent system reduces to three transmission lines system as in [4]. The expressions for characteristics admittances and impedances of transmission lines in the SPICE model (Figure 1.a,b) are readily found from equation (2) and (3) and given by,

$$Y_\pi^1 = \frac{Y_{\pi 1}(R_c - 1)}{R_c - R_\pi}, \quad Y_c^1 = \frac{Y_{c1}(1 - R_\pi)}{R_c - R_\pi}$$

$$Y_\pi^2 = \frac{R_c Y_{c2}(1 - R_\pi)}{R_c - R_\pi} \quad Y_c^2 = \frac{R_\pi Y_{\pi 2}(R_c - 1)}{R_c - R_\pi}$$

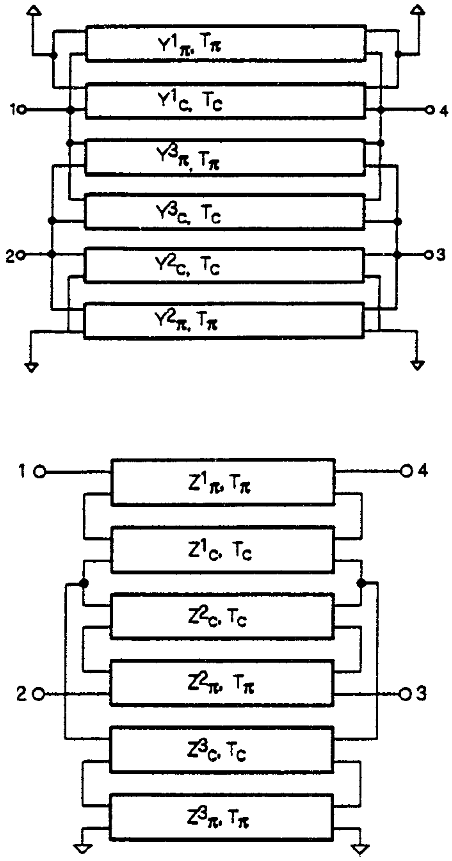

Fig. 1. a.) Spice model for asymmetric coupled lines based on four port admittance matrix and b.) impedance matrix.

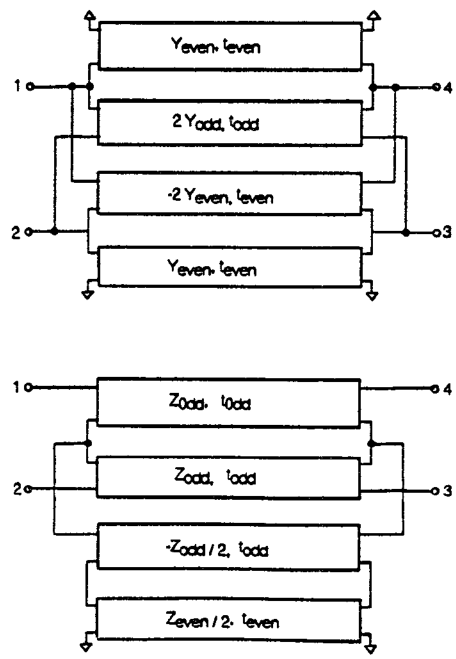

Fig. 2. a.) Spice model for symmetrical coupled lines based on four port admittance matrix and b.) impedance matrix.

$$Y_\pi^3 = \frac{Y_{\pi 1}}{R_c - R_\pi} \quad Y_c^3 = \frac{-Y_{c1}}{R_c - R_\pi} \tag{4}$$

$$Z_\pi^1 = \frac{Z_{\pi 1} R_c}{R_c - R_\pi}(1 - R_\pi), \quad Z_c^1 = \frac{Z_{c1} R_\pi}{R_c - R_\pi}(R_c - 1)$$

$$Z_\pi^2 = \frac{Z_{\pi 2}}{R_c - R_\pi}(1 - R_\pi), \quad Z_c^2 = \frac{Z_{c2}}{R_c - R_\pi}(R_c - 1)$$

$$Z_\pi^3 = \frac{Z_{\pi 1} R_c R_\pi}{R_c - R_\pi}, \quad Z_c^3 = -\frac{Z_{c1} R_c R_\pi}{R_c - R_\pi} \tag{5}$$

The characteristics admittance matrix parameters of symmetrical three coupled lines is found in a similar manner in terms of the line mode admittances of three lines, the mode voltages and current ratios and propagation constants for the normal modes [9] and is given by

$$[Y_c] = \begin{bmatrix} \frac{Y_{a1}(R_{v1}-R_{v2})-Y_{a2}R_{v2}+Y_{a3}R_{v1}}{2(R_{v1}-R_{v2})} & \frac{Y_{a2}-Y_{a3}}{R_{v1}-R_{v2}} & \frac{Y_{a1}(R_{v2}-R_{v1})-Y_{a2}R_{v2}+Y_{a3}R_{v1}}{2(R_{v1}-R_{v2})} \\ \frac{-R_{v1}R_{v2}Y_{b2}+R_{v1}R_{v2}Y_{b3}}{2(R_{v1}-R_{v2})} & \frac{R_{v1}Y_{b2}-R_{v2}Y_{b3}}{(R_{v1}-R_{v2})} & \frac{-R_{v1}R_{v2}Y_{b2}+R_{v1}R_{v2}Y_{b3}}{2(R_{v1}-R_{v2})} \\ \frac{-(R_{v1}-R_{v2})Y_{c1}-R_{v2}Y_{c2}+R_{v1}Y_{c3}}{2(R_{v1}-R_{v2})} & \frac{Y_{c2}-Y_{c3}}{R_{v1}-R_{v2}} & \frac{-(R_{v2}-R_{v1})Y_{c1}-R_{v2}Y_{c2}+Y_{c3}R_{v1}}{2(R_{v1}-R_{v2})} \end{bmatrix} \tag{6}$$

Here γ_a, γ_b, γ_c are the propagation constants for the normal modes, Y_{a1},Y_{a2},.. etc. are the characteristic admittances for three lines for modes a,b and c and the corresponding voltage and current eigenvector matrices. For three symmetric coupled lines $Y_{a1} = Y_{a3}$, $Y_{b1} = Y_{b3}$ and $Y_{c1} = Y_{c3}$. This matrix is readily realized leading to the transmission line network with fifteen transmission lines with six ports whose characteristic admittances are given by

transmission lines between : ports 1 and 4,

$$Y_{a1}, \quad \frac{-Y_{a2}}{R_{v1} - R_{v2}}, \quad \frac{Y_{a3}}{R_{v1} - R_{v2}}$$

ports 2 and 5,

$$\frac{Y_{b2}(R_{v1} + R_{v1}R_{v2})}{R_{v1} - R_{v2}}, \quad \frac{-Y_{b3}(R_{v2} + R_{v1}R_{v2})}{R_{v1} - R_{v2}}$$

ports 3 and 6,

$$Y_{c1}, \quad , \frac{-Y_{c2}}{R_{v1} - R_{v2}}, \quad \frac{Y_{c3}}{R_{v1} - R_{v2}}$$

terminal pairs 1, 2 and 4, 5

$$\frac{Y_{a2}}{R_{v1} - R_{v2}}, \quad \frac{-Y_{a3}}{R_{v1} - R_{v2}}$$

terminal pairs 2, 3 and 5, 6

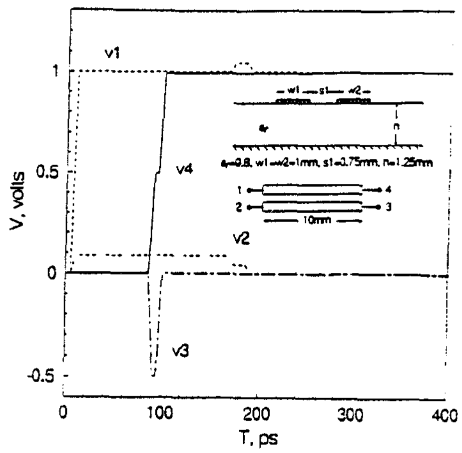

Fig. 3. Step response of the symmetric coupled microstrip four port

$$\frac{-R_{v1}R_{v2}Y_{b2}}{2(R_{v1} - R_{v2})}, \quad \frac{R_{v1}R_{v2}Y_{b3}}{2(R_{v1} - R_{v2})}$$

terminal pairs 1, 3 and 5, 6

$$\frac{Y_{a1}}{2}, \quad \frac{-R_{v2}Y_{a2}}{2(R_{v1} - R_{v2})}, \quad \frac{R_{v1}Y_{a3}}{2(R_{v1} - R_{v2})}$$

The electrical length of these transmission lines respectively corresponds to the electrical length of the three modes.

IV. Results and discussion

In order to demonstrate the usefulness and the versalities of these models, the frequency and time domain response of coupled microstrip structures is presented. The time domain response of the circuit in Figure 2a based on SPICE simulation is shown in Figure 3. The time domain step response of the same asymmetric coupled microstrip four port as that in ref. [1] is shown in Figure 4 and validates the accuracy of the model.

Figure 5 shows the frequency response of coupled two section filter simulated by Libra and using the coupled line model given in Figure 1a.

A general three symmetrical coupled lines step response is shown in Figure 6. The input and output ports are terminated by 50 ohms impedances except port 5 which drives a high impedance CMOS inverter. The response shows coupling and crosstalk effects between the lines.

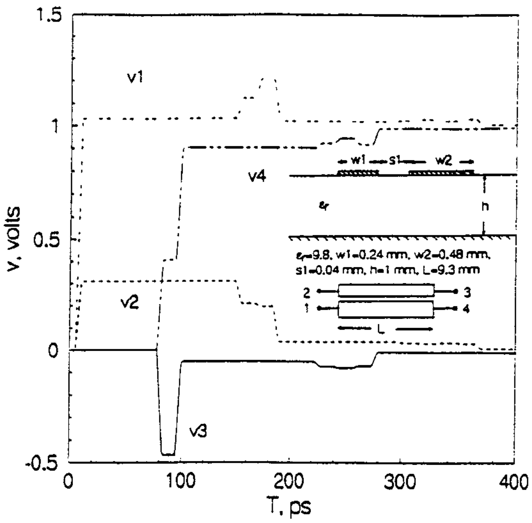

Fig. 4. Step response of the asymmetric coupled microstrip four port.(50–ohms termination)

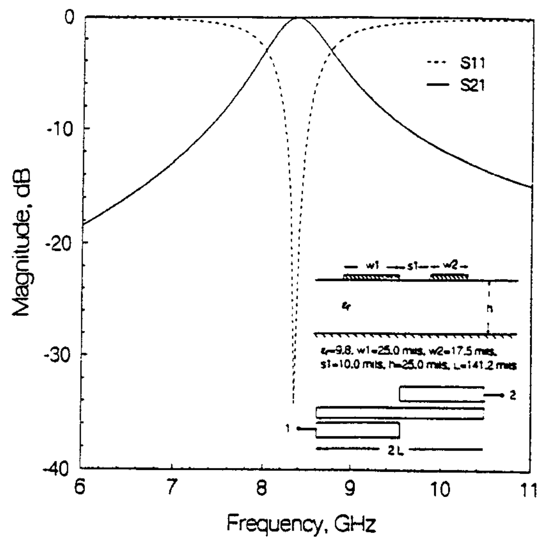

Fig. 5. Frequency response of asymmetric coupled microstrip filter.

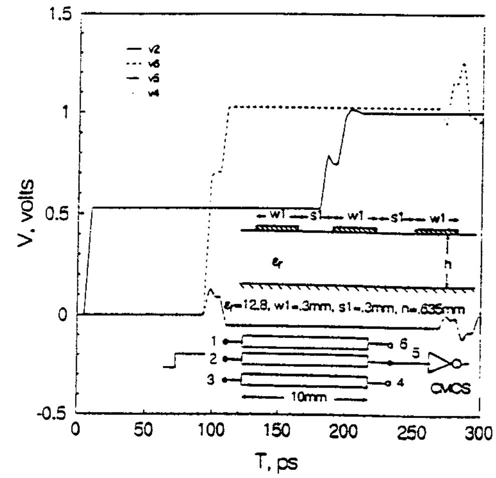

Fig. 6. Step response of the three symmetric coupled microstrip six ports.(50–ohms termination at port 1,2,3,4,6)

V. conclusion

In conclusion, a new configuration-oriented SPICE model for multiconductor inhomogeneous lines is presented. The model is compatible with lossy, dispersive systems and should be quite helpful in the frequency and time domain simulation and design of multiconductor coupled systems.

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